the perfect tube

student activity

Mathematics B Education Program

Name: ______________________________

Teacher: ______________________________

School: ______________________________

WhiteWater World runs on tubes, from the simple donut rings of the Cave of Waves to the figure-of-eight tubes of The Temple of Huey and the extreme Cloverleaf rafts of The Green Room and The Rip. There's a large volume of compressed air that goes into keeping WhiteWater World afloat – but exactly how much? You're about to find out!

Syllabus Links

Basic knowledge and procedures - metric measurement including measurement of mass, length, area and volume in practical contexts

Maintaining mathematical procedures - practical applications of volume and surface area of regular shapes

Optimisation using derivatives - investigate situations finding optimal quantities and/or optimal costs such as the optimal use of materials used for the manufacture of various containers of simple shapes

Rates of change - determine the instantaneous rate of change of a variable with respect to another variable in life-related situations given the mathematical model; such as the rate of change of the surface area of an object with respect to volume

Equipment

Student activity sheets, pens/pencils, graphics calculators, metre ruler or tape measure (optional)

Activities with this symbol may be completed while you're having lunch or after you leave WhiteWater World.

Standard Achieved

Knowledge & Procedures

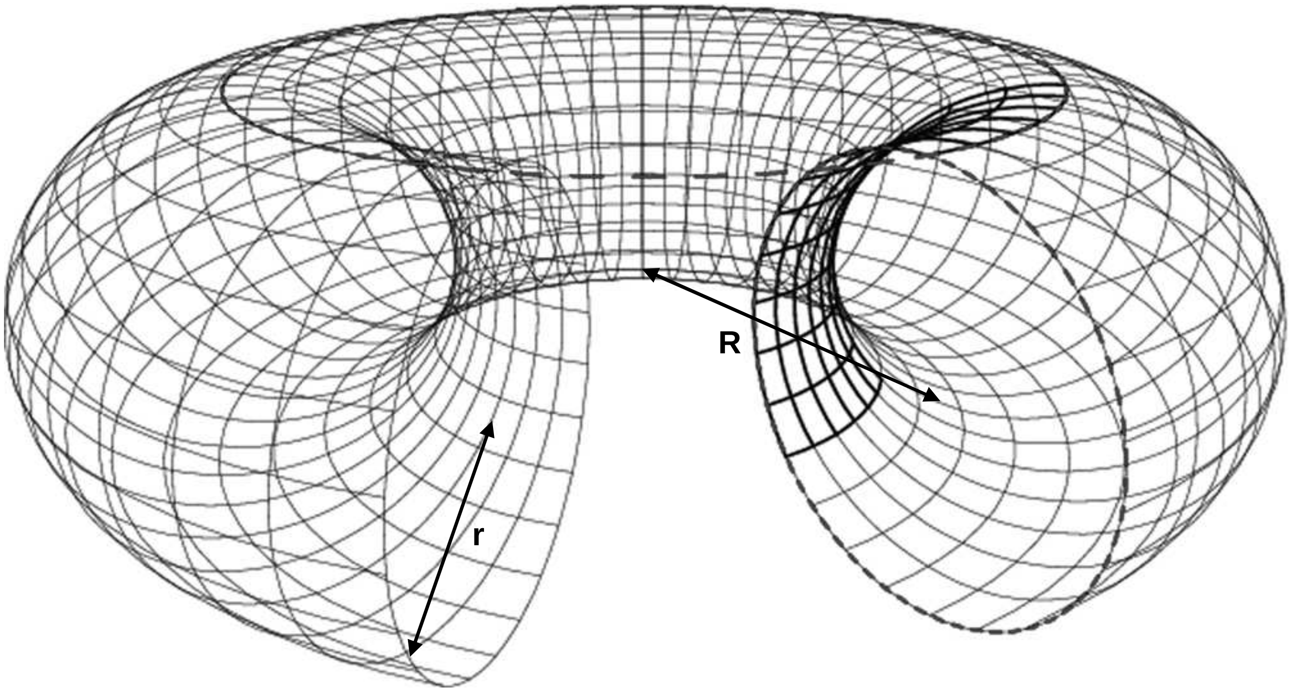
Communication & Justification

Inflatable tubes are an important part of many of the rides at WhiteWater World. The simplest single-person tube is in the shape of a donut. In mathematics, this 3D shape is called a **torus**. The two-person figure-of-8 tube at The Temple of Huey is in the shape of two toruses joined together, so it's called a **double torus.**

"Torus" comes from the Latin word for "bulge" and was first used to describe the moulding around the base of a column. A torus can be thought of as a surface of revolution obtained by rotating a circle around an axis in the plane of the circle but not intersecting the circle.

Volume of a Torus

A torus can be thought of as a cylinder twisted into a circle, so both ends touch. This is a useful way of considering it when calculating the volume of the torus. The volume of the torus is the same as the volume of the cylinder, its cross-sectional area multiplied by its length.



If r is the radius of the cylinder, its cross-sectional area is πr^2. If R is the radius of the torus, the length of the cylinder is the circumference of the torus through the centre of the cylinder, $2\pi R$.

Question 1

Use the above information to write the formula for the volume of a torus. Simplify as much as possible.

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Volume of a torus: V = ____________________________

where r = radius of cylinder
R = radius of torus

Surface Area of a Torus

Like the volume, the surface area of a torus is simply the surface area of the cylinder before it's twisted into a circle (without the top and bottom, of course). That is, it's the circumference of the torus through the centre of the cylinder ($2\pi R$) multiplied by the circumference of the cylinder ($2\pi r$).

Question 2

Use the above information to write the formula for the surface area of a torus. Simplify as much as possible.

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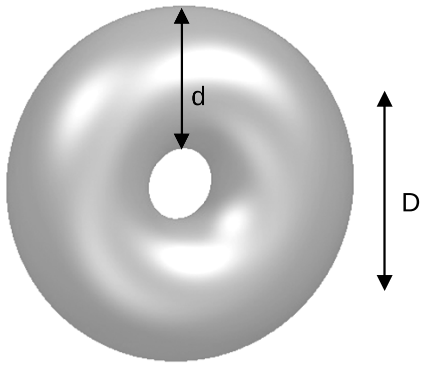
Surface area of a torus: SA = ______________________________

where r = radius of cylinder
R = radius of torus

Volume of a donut tube at The Cave of Waves

Question 3



Find three donut tubes of different sizes at The Cave of Waves and use a tape measure or ruler to measure the radius of the cylinder (*r*) and the radius of the torus (*R*) for each. It will be easier and more accurate to instead measure the diameter of the cylinder (*d*) and the diameter of the torus (*D*) and then divide each by 2 to obtain the respective radii.

If you look carefully, you will note that the cylinder is not perfectly circular. Ignore this discrepancy.

Record your results in the table below. Use the space on the next page for your volume and surface area calculations.

	d (cm)	D (cm)	r (cm)	R (cm)	Volume	Surface Area
Donut Tube 1						
Donut Tube 2						
Donut Tube 3						

If you don't have access to a ruler or tape measure, use these dimensions:

d (cm)	D (cm)
28	67
30	70
35	80

Question 4

Use the formulae you derived in Questions 1 and 2 to calculate the volume and surface area of each of the tubes you measured in Question 3.

Convert your answers to the most convenient units using the following conversion factors:

$1 \times 10^6\ cm^3 = 1 \times 10^3\ L = 1m^3$

$1 \times 10^4\ cm^2 = 1m^2$

Fill in your answers in the spaces in Question 3.

Question 5

The cost of manufacturing donut tubes depends directly on the amount of plastic used – that is, on the surface area of the tube. Dreamworld engineers want a tube which maximises the volume to surface area ratio. They need to work out whether they should focus on the radius of the cylinder (*r*), the radius of the torus (*R*), or both.

Using the equations for volume ($V = 2\pi^2 Rr^2$) and surface area ($SA = 4\pi^2 Rr$), create an expression for the ratio of volume to surface area, $\frac{V}{SA}$. Simplify your expression.

$$\frac{V}{SA} =$$

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Use your result to comment on the effect of R and r on the ratio of volume to surface area.

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Treating R as constant, write expressions for the rate of change of Volume with respect to r, $\frac{dV}{dr}$, and Surface Area with respect to r, $\frac{dSA}{dr}$. Simplify your expressions.

$$\frac{dV}{dr} =$$

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$$\frac{dSA}{dr} =$$

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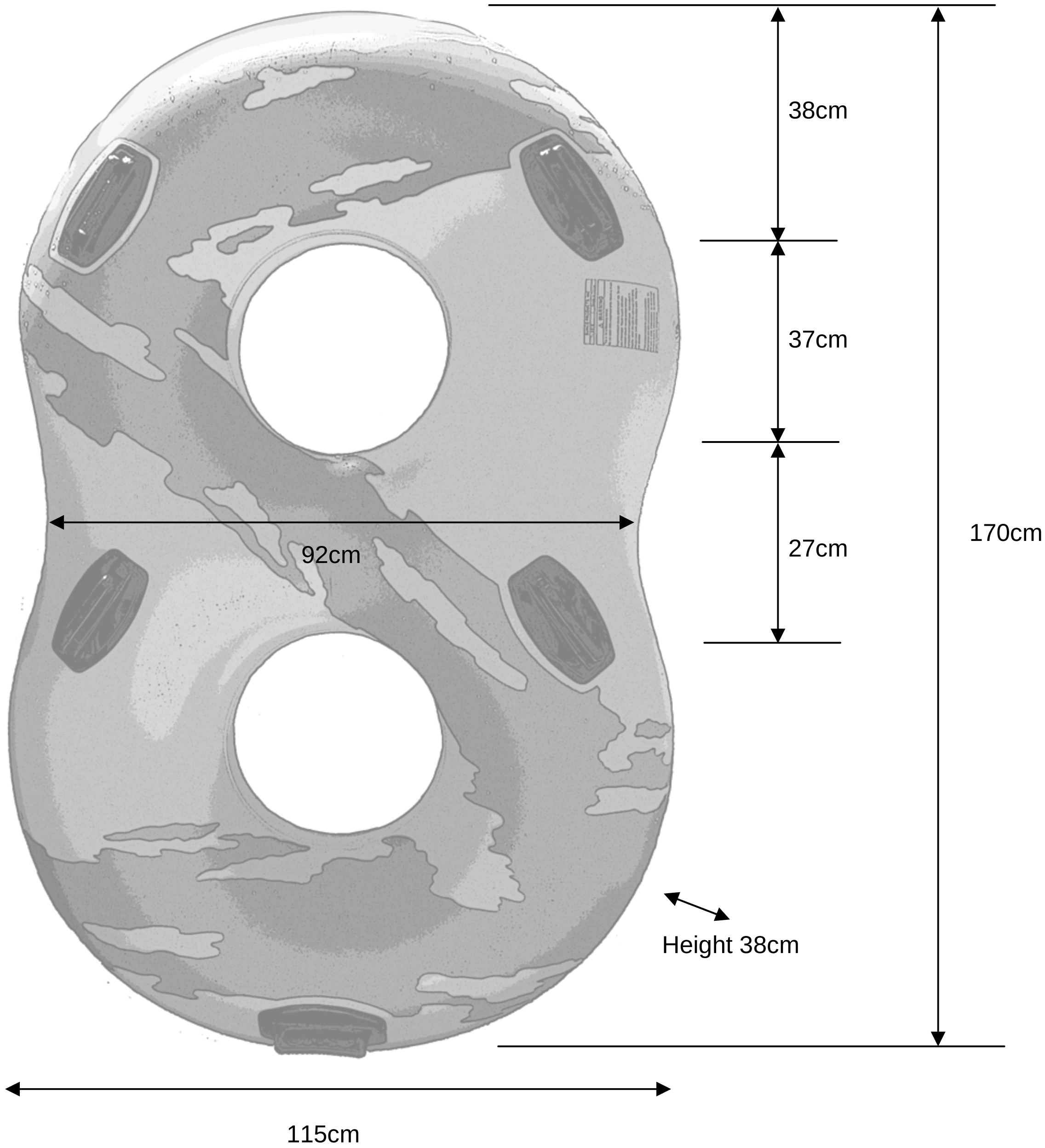
Volume of a figure-of-8 tube at The Temple of Huey

Question 6

Find a two-person figure-of-8 tube at The Temple of Huey and use a tape measure or ruler to measure its dimensions. Again, it will be easier and more accurate to measure diameters and then divide by 2 to obtain radii. Use the picture below to mark your measurements.

If you don't have access to a ruler or tape measure, use these dimensions:



Use the following procedure to calculate the volume of the figure-of-8 tube.

Question 7

A figure-of-8 tube is a double torus, so to calculate its volume, you first need to find the volumes of the two toruses involved. Assume that they are the same size.

From your measurements above, determine the radius of the cylinder (*r*) and the radius of the torus (*R*).

r = ____________ cm

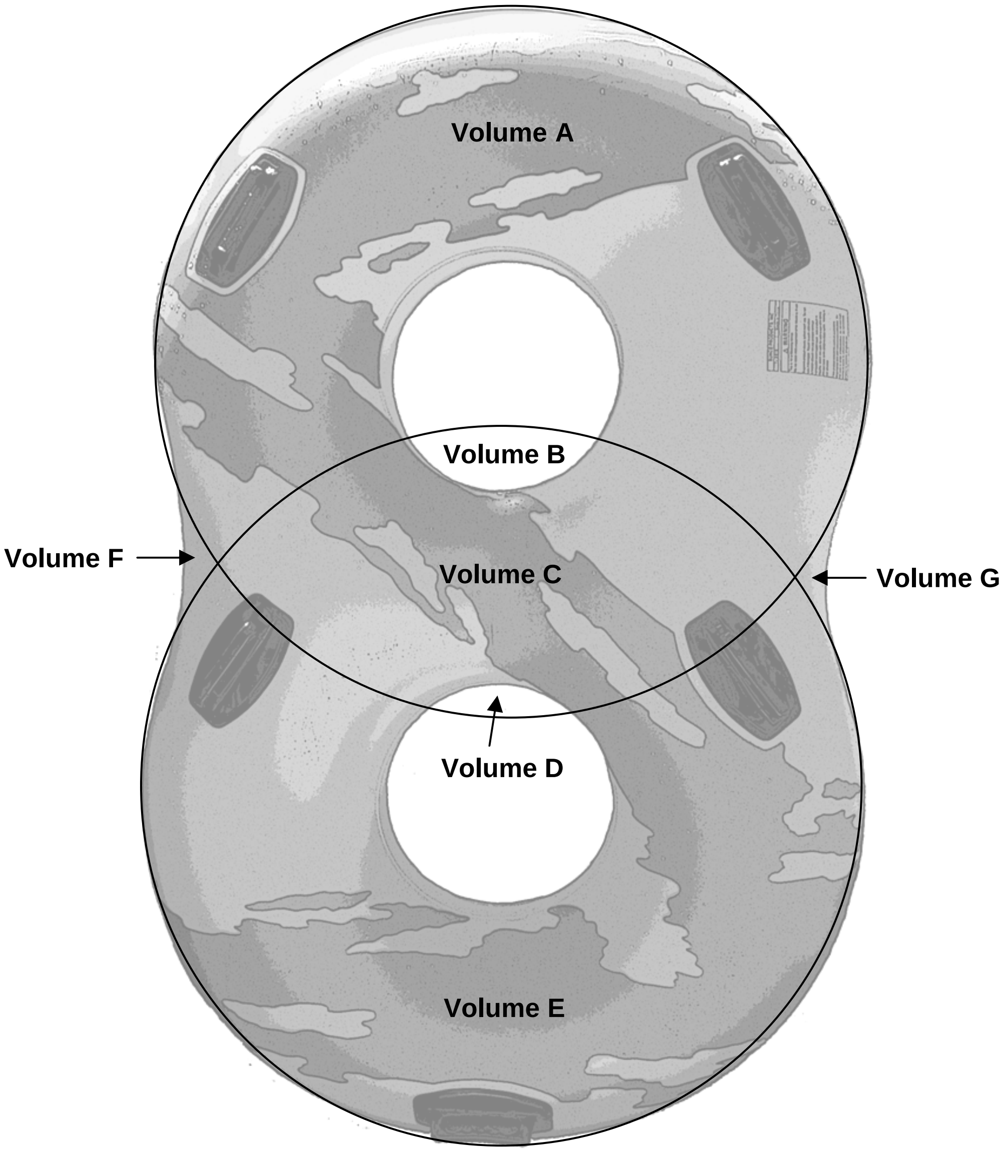
R = ____________ cm

Question 8

Using the radii from Question 6, calculate the volume of one torus. Leave your answer in cm^3 for now.

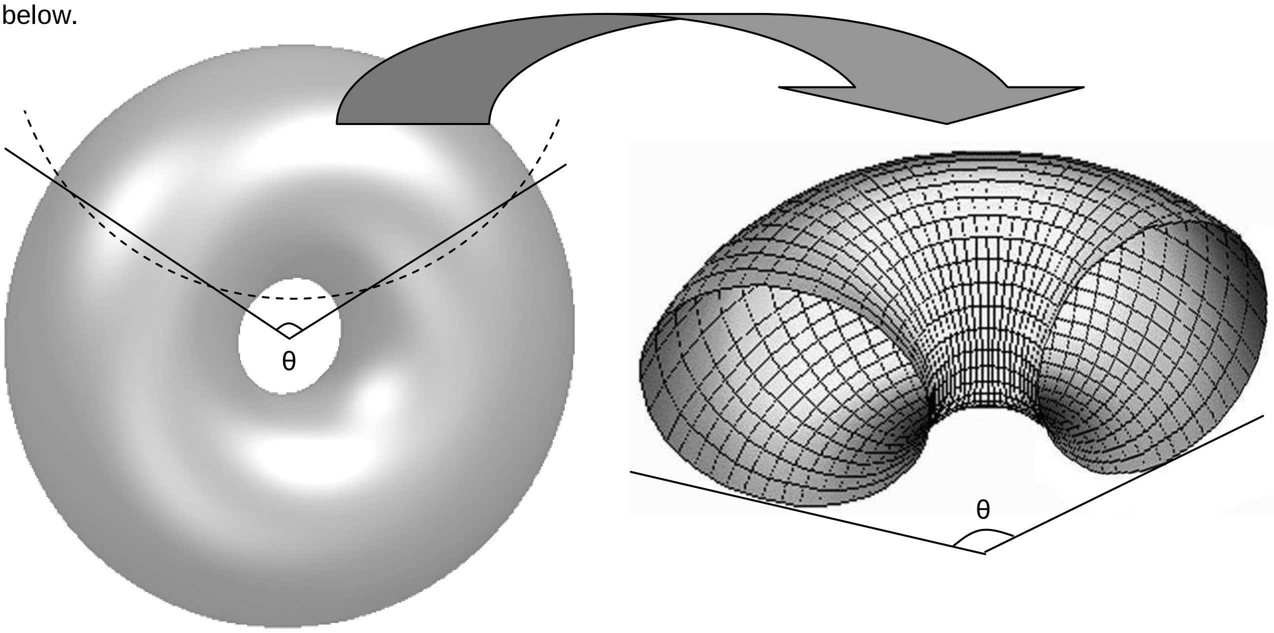
V = ____________ cm^3

The volume of a double torus is not simply double the volume of a torus since there is a section of overlap. Overlaying the two toruses as in the diagram below, we can label the area of overlap Volume C. We then need to subtract Volumes B and D and add Volumes F and G. For simplicity here, we will make the assumption that the volume subtracted for B and D is equal to the volume added for F and G, allowing us to ignore these sections.



Question 9

To calculate Volume C, we make the approximation that one torus cuts the other in a straight slice, as shown below.



Using a protractor, angle θ can be shown to be about 115°. We can treat Volume C as a slice of the donut, representing a fraction of $\frac{115}{360}$ of the total volume of the torus.

Calculate Volume C.

V_C = ___________ cm³

Question 10

The volume of the double torus is double the volume of the single torus calculated in Question 7, less Volume C calculated in Question 8.

Calculate the volume of the double torus.

V = ___________ cm³

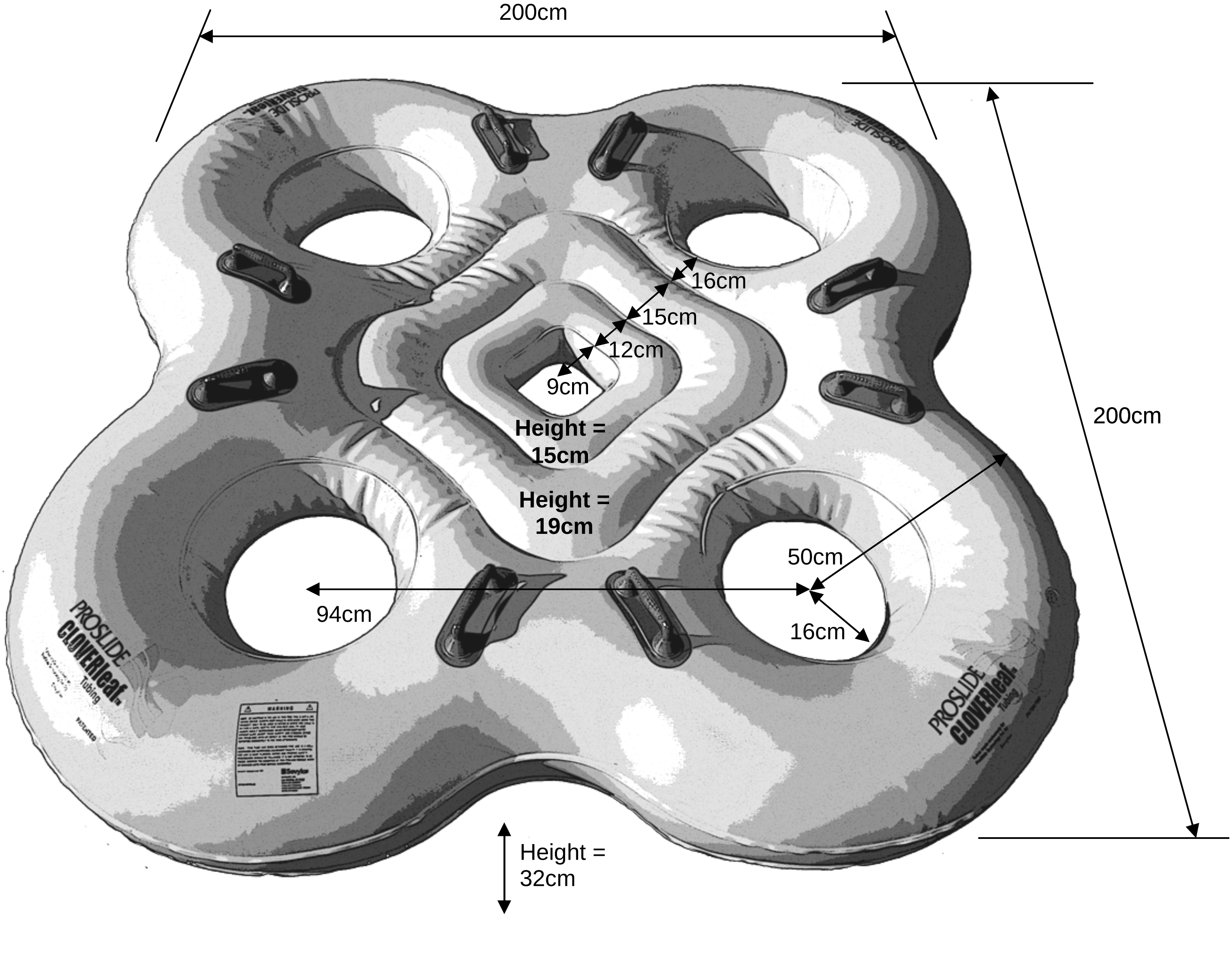
Question 11

Convert the volume of the double torus to litres (L), using the conversion $1 \times 10^3 cm^3 = 1L$.

V = ___________ L

Volume of a Cloverleaf Tube

The shape of the cloverleaf tubes used in The Green Room and The Rip is a little more complex. Approximate dimensions are shown below.



Question 12

Calculate the volume of a cloverleaf tube.

Notice that a cloverleaf tube is simply two figure-of-8 tubes joined together, so use the steps that you learnt in Questions 6 through 10 to first find the volume of a figure-of-8 tube. Then use a similar method to find the volume of two figure-of-8 tubes joined together to form a cloverleaf tube.

Option 1: Ignore the effect of the pattern in the middle section.

Option 2 (Extension): Include the effect of the pattern in the middle section. Note that this can be thought of as two cylinders bent into a square shape. As with a torus, the volume is the same as the volume of the cylinder before it was bent.

Convert your final answer to litres (L).

Question 13

Use the volumes that you have calculated for each of the inflatable tubes to estimate the volume of compressed air used to inflate all of the tubes at WhiteWater World.

Tube	Volume (L)	Number in use	Total Volume (L)
Donut Tube 1		30	
Donut Tube 2		30	
Donut Tube 3 (largest)		50	
Figure-of-8 Tube		50	
Cloverleaf Tube		30	
		Total	

Convert the final volume to m^3 using the conversion 1×10^3 L = $1m^3$.

Total Volume = ___________ m^3