student activity

WHITEWATER WORLD
BY DREAMWORLD

Mathematics B Education Program

Name: _____

Teacher: _____

School: _____

Water waves are one of the most inspiring examples of periodic functions in action – but it's nearly impossible to stay still for long enough to measure them accurately at the beach. This is why the Cave of Waves is the perfect place to learn about periodic functions. There's certainly no shortage of waves, but it's as safe as a swimming pool!

Syllabus Links

Periodic functions and applications - definition of a periodic function, the period and amplitude. For instance:

- find the period, amplitude and frequency of trigonometric functions which are used to model phenomena such as wave motion
- plot the water heights at a specified point as waves pass over a certain time period

Basic knowledge and procedures - metric measurement including measurement of mass, length, area and volume in practical contexts

Maintaining mathematical procedures - practical applications of volume and surface area of regular shapes

Maintaining mathematical procedures - interpretation and drawing of scale drawings and plans

Introduction to integration - Determine the volume of water that could be contained in a pool or tank of given length and parabolic cross-section

Equipment

Student activity sheets, pens/pencils, short ruler, graphics calculators, stopwatches, metre ruler or tape measure (optional)

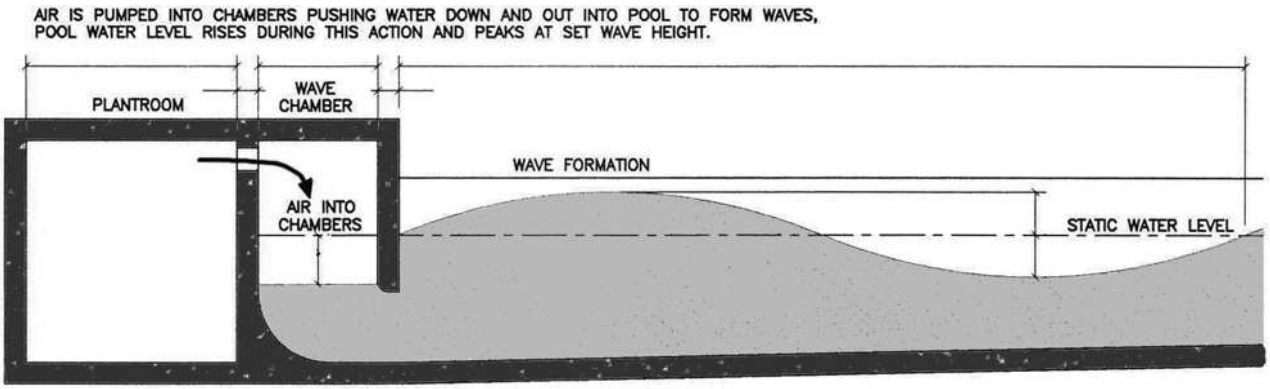
Activities with this symbol may be completed while you're having lunch or after you leave WhiteWater World.

Standard Achieved

Knowledge & Procedures ☐ Communication & Justification ☐

The Periodic Function of a Wave

Water waves are a great way to learn about periodic functions. In the Cave of Waves, giant fans pump air in and out of specially designed chambers to produce perfect waves. In this activity you'll measure the period and amplitude to find the equation of the waves.



The Equation of a Periodic Function

Periodic functions are of the form $y = A\sin[B(x-C)]+D$.

Where:

A is the amplitude (half the difference in height between the highest and lowest points)

B relates to the period, where the period is the time it takes for one complete cycle and Period = $\frac{2\pi}{B}$ (radians) or $\frac{360}{B}$ (degrees). Hence, when the period is known, B can be determined by calculating $\frac{2\pi}{Period}$ (radians) or $\frac{360}{Period}$ (degrees).

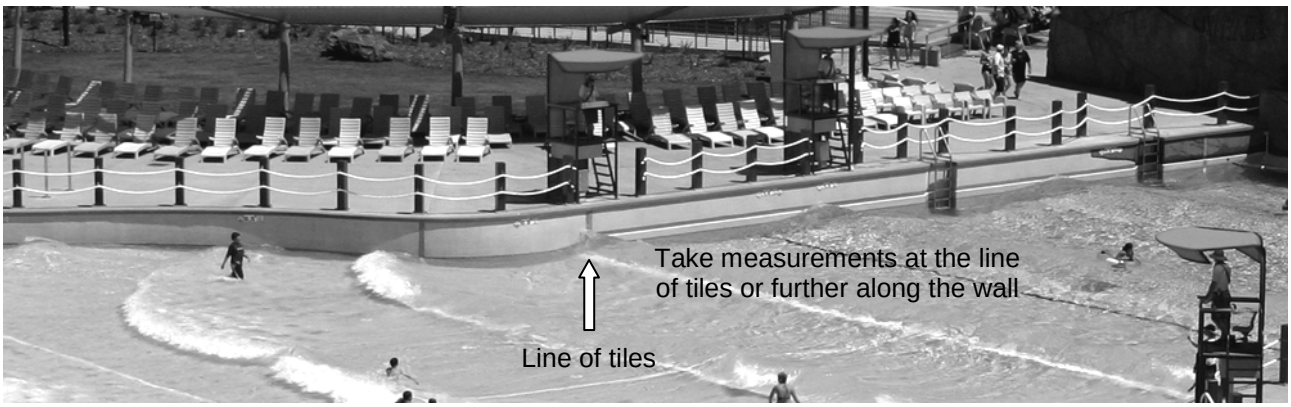
C measures the horizontal or phase shift.

D measures the vertical shift.

Because the placement of the y-axis is arbitrary, we can ignore C here, and simplify our equation to $y = A\sin Bx + D$.

To determine our equation, we must find A, B and D.

Find yourself a place in the Cave of Waves where you can take measurements comfortably. It will be helpful to stay close to the side wall, and the line of tiles along the wall may prove to be a useful reference point. Choose a place before the waves break.



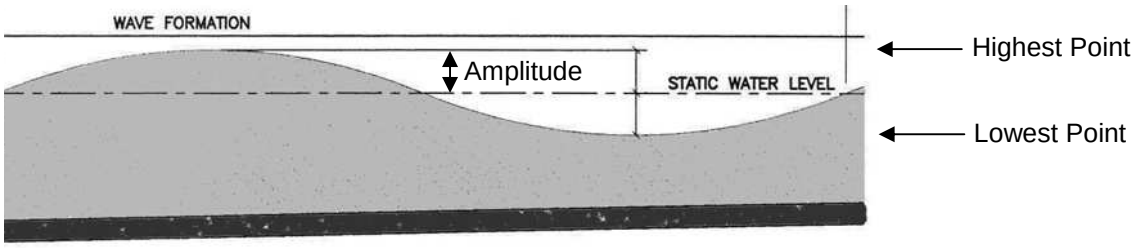
Question 1

Use a ruler or tape measure to determine the amplitude and vertical shift of the waves. First measure the height of the highest point that the waves reach above the bottom of the pool. For accuracy, repeat your measurement three times and find an average.

Highest point measurement 1 _____ cm
Highest point measurement 2 _____ cm
Highest point measurement 3 _____ cm
Highest point average _____ cm

Lowest point measurement 1 _____ cm
Lowest point measurement 2 _____ cm
Lowest point measurement 3 _____ cm
Lowest point average _____ cm

If you don't have a ruler or tape measure handy, use the line of tiles on the northern side of pool between the 0.5m and 0.6m marks. The top of the horizontal tile line is 60cm from the bottom of pool. 13 vertical tiles reach a total of 78cm in height. Use this to calculate the height of each tile, and hence the highest and lowest points of the waves in cm.



The amplitude is half of the difference between the high and low points. Calculate this value.

_____

_____

_____

_____

A = _____ cm

The static water level is the average of the high and the low points. This is the value of the vertical shift, D, in the equation. Calculate this value.

_____

_____

_____

_____

D = _____ cm

Question 2

Use a stopwatch to measure the period of the wave. The period is the time that elapses between one wave and the next. Choose a reference point on the wall, and measure the length of time between successive peaks or successive troughs. For accuracy, repeat your measurement three times and find an average.

Period measurement 1 _____ s

Period measurement 2 _____ s

Period measurement 3 _____ s

Period average _____ s

To calculate B in the formula, we use the equation derived above, $B = \frac{2\pi}{Period}$. Calculate this value.

_____

_____

_____

_____

B = _____

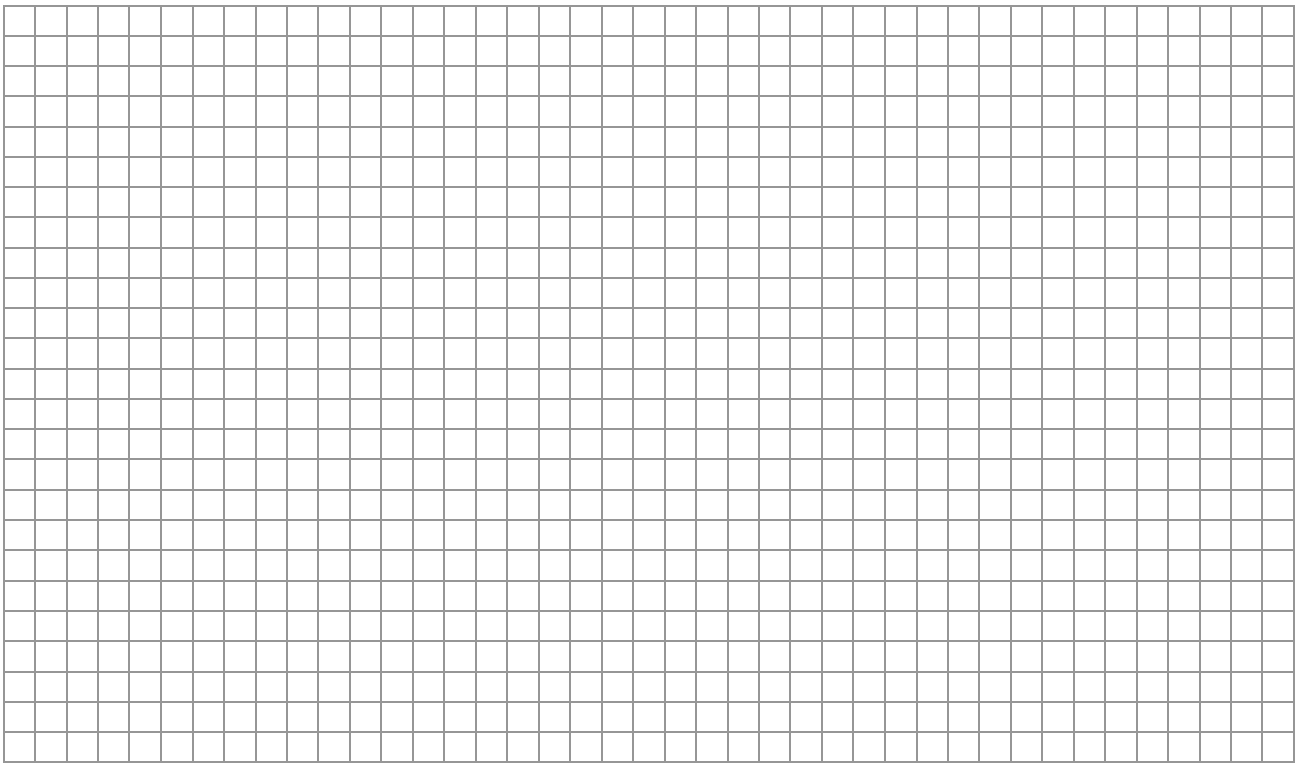
Question 3

Use the values you have calculated for A, B and D to complete the formula $y = A\sin Bx + D$.

_____

Question 4

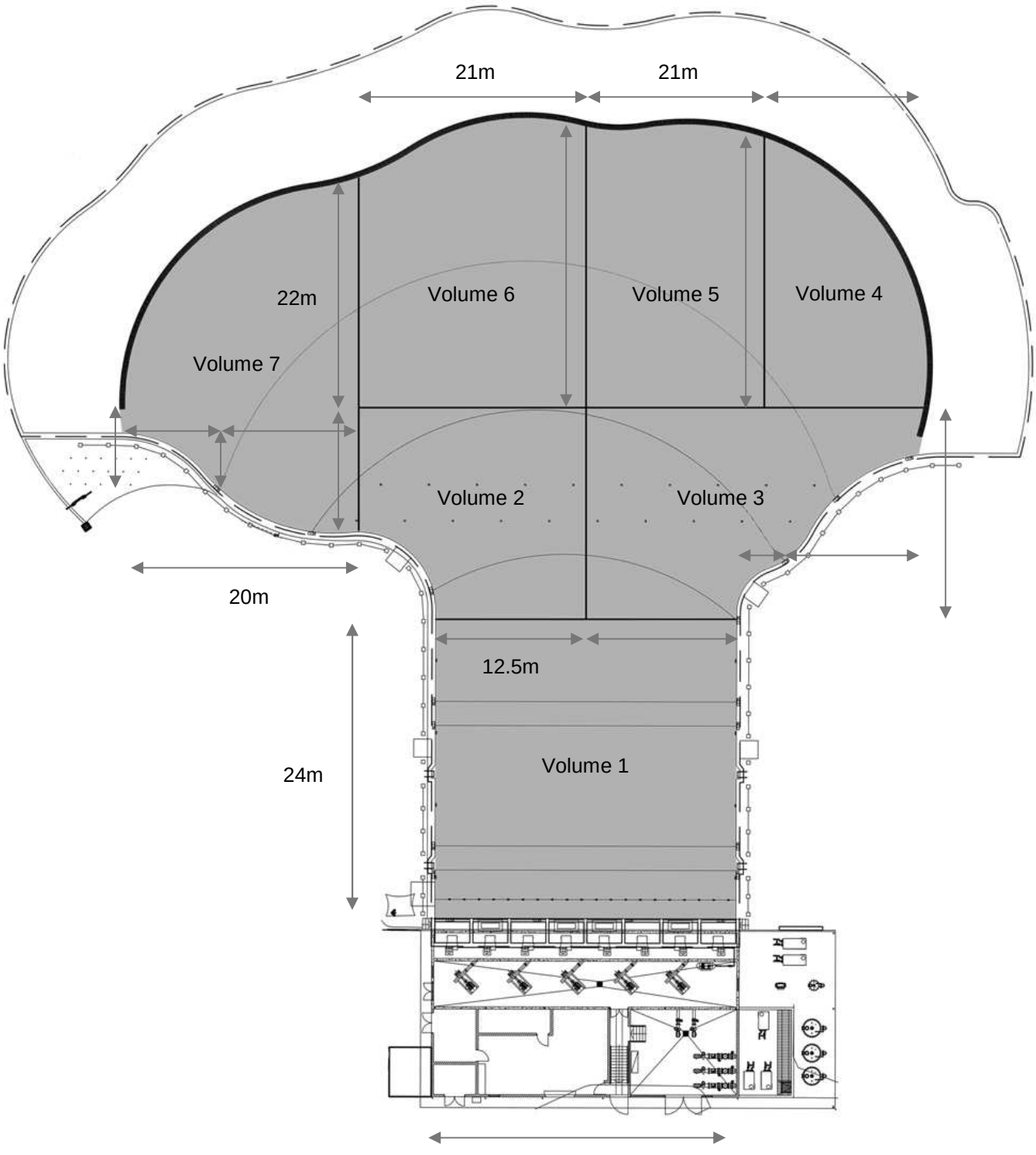
Draw a sketch of one cycle of your function.



The Volume of Water in the Cave of Waves

Engineers need to know precisely how much water is contained in the cave of waves in order to design the structure to handle the stress, to set the wave machine to generate waves correctly and to ensure that chlorine additions are in the right proportion. In this activity you will measure key dimensions of the wave pool and calculate its volume.

Because the pool is a complex shape, it will be easier to first break it down into simpler shapes. The lines of white tiles along the floor of the pool conveniently divide it into seven sections which will serve as a useful starting point for calculating the volume.



Scale 1:500

Question 5

Twelve dimensions are missing from the diagram on the previous page. Use a ruler and the scale of the drawing to calculate these dimensions. Mark them on the diagram.

Working Space:

_____

_____

_____

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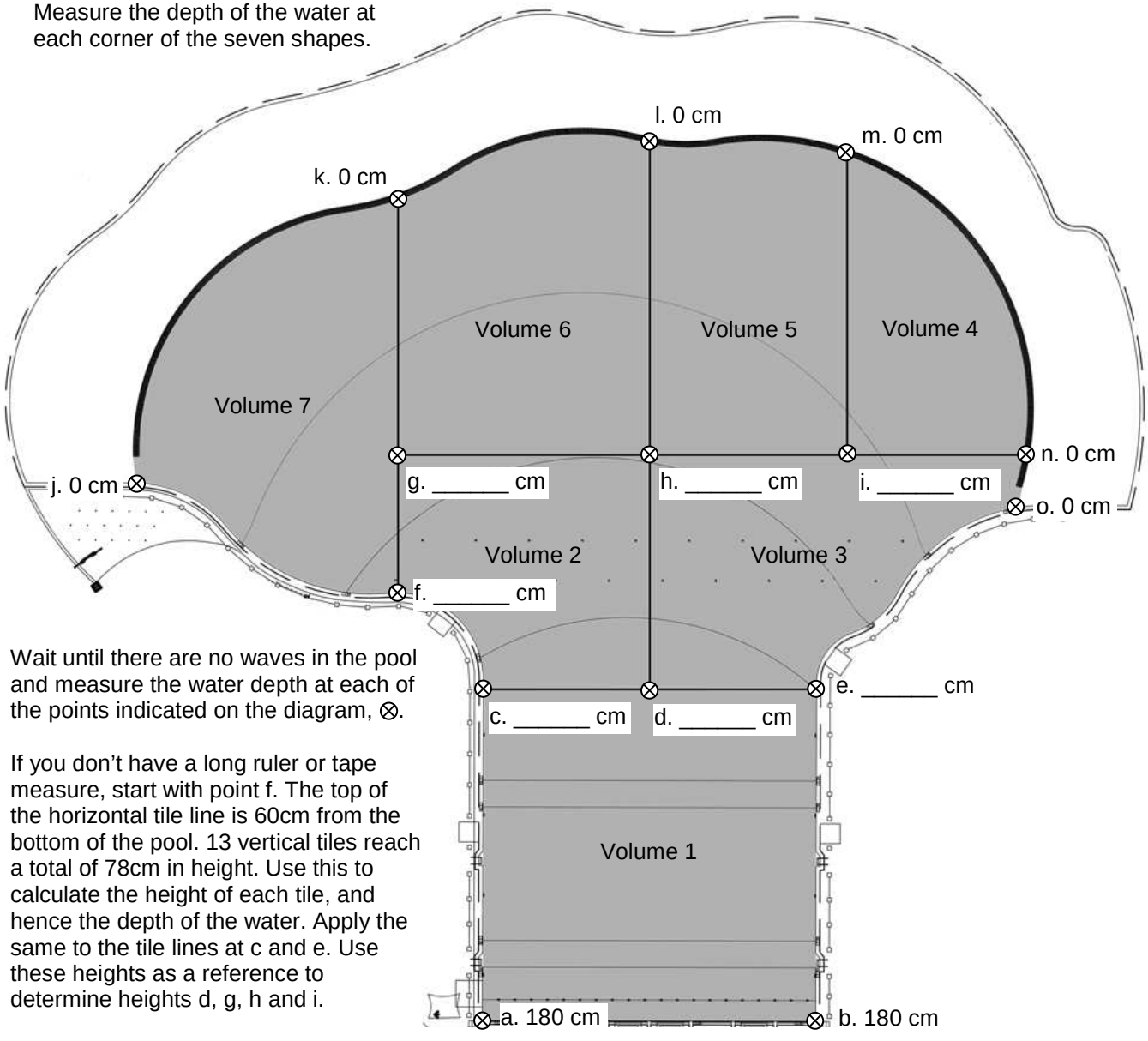
Question 6

Measure the depth of the water at each corner of the seven shapes.

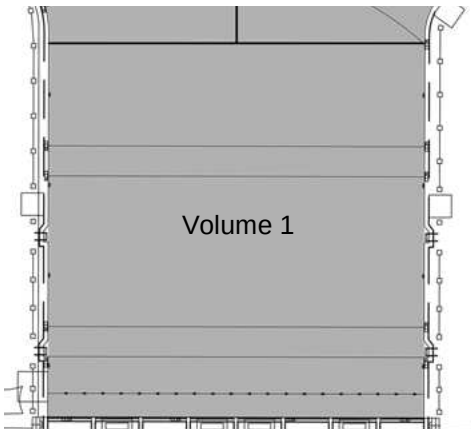


Wait until there are no waves in the pool and measure the water depth at each of the points indicated on the diagram, ⊗.

If you don't have a long ruler or tape measure, start with point f. The top of the horizontal tile line is 60cm from the bottom of the pool. 13 vertical tiles reach a total of 78cm in height. Use this to calculate the height of each tile, and hence the depth of the water. Apply the same to the tile lines at c and e. Use these heights as a reference to determine heights d, g, h and i.

Question 7

Calculate Volume 1. First mark on the diagram to the right each of the relevant dimensions from pages 5 and 6.



The volume of a prism is the product of the area of its base and its height. To simplify our calculations, we will take the height as the average of the heights at the four extremities.

Calculate the average height from your four measurements.

_____

_____

_____

_____

Average height = _____ cm

Now calculate the area of the base from its length and width. For consistency, convert to cm first.

_____

_____

_____

Area of base = _____ cm^2

Use the average height and the area of the base to calculate Volume 1.

_____

_____

_____

Volume 1 = _____ cm^3

Convert your answer to m^3 using the conversion 1×10^6 $cm^3 = 1$ m^3.

_____

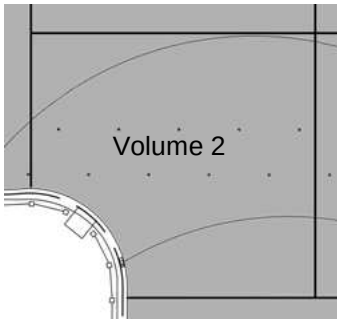
_____

_____

Volume 1 = _____ m^3

Question 8

Calculate Volume 2. First mark on the diagram to the right each of the relevant dimensions from pages 5 and 6.



Calculate the average height from your five measurements.

_____

_____

_____

_____

Average height = _____ cm

Now calculate the area of the base. You will first need to find the area of the rectangle, then subtract the section that is missing. Note that this is the shape of a quarter of a circle $\left(A = \frac{1}{4}\pi r^2\right)$. For consistency, convert to cm first.

_____

_____

_____

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Area of base = _____ cm^2

Use the average height and the area of the base to calculate Volume 2.

_____

_____

_____

Volume 2 = _____ cm^3

Convert your answer to m^3 using the conversion 1×10^6 $cm^3 = 1$ m^3.

_____

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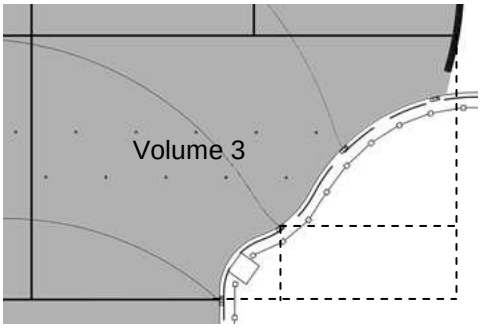
_____

Volume 2 = _____ m^3

Question 9



Calculate Volume 3. First mark on the diagram to the right each of the relevant dimensions from pages 5 and 6.

Calculate the average height from your six measurements.

_____

_____

_____

_____

Average height = _____ cm

Now calculate the area of the base. You will first need to find the area of the rectangle, then subtract the section that is missing. It's a little more complicated than it was for Volume 2. You can treat the missing volume here as two quarter-circles and a rectangle. For consistency, convert to cm first.

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_____

Area of base = _____ cm^2

Use the average height and the area of the base to calculate Volume 3.

_____

_____

Volume 3 = _____ cm^3

Convert your answer to m^3 using the conversion 1×10^6 $cm^3 = 1$ m^3.

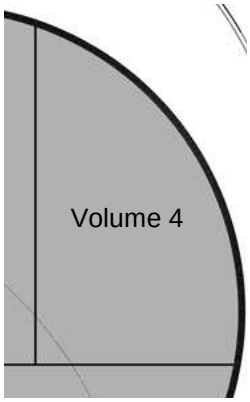
_____

_____

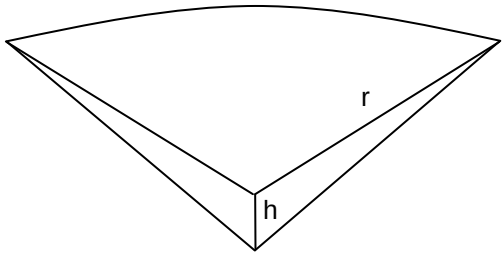
Volume 3 = _____ m^3

Question 10

Calculate Volume 4. First mark on the diagram to the right each of the relevant dimensions from pages 5 and 6.



Since Volume 4 has zero depth along its outer edge and its area is a section of a circle, we can find its volume more accurately by approximating it as a quarter of a cone.



We will take the radius of the cone to be the average of the two radii measured (the length and width that you have marked on the diagram).

Calculate the average radius from your two measurements. For consistency, convert to cm first.

_____

_____

Average radius = _____ cm

The volume of a cone is $V = \frac{1}{3}\pi r^2 h$ where r is the radius and h is the height.

Write the formula for the volume of a quarter of a cone.

V = _____

Now calculate Volume 4 using your measurements.

_____

_____

_____

_____

_____

_____

_____

Volume 4 = _____ cm^3

Convert your answer to m^3 using the conversion 1 x 10^6 cm^3 = 1 m^3.

_____

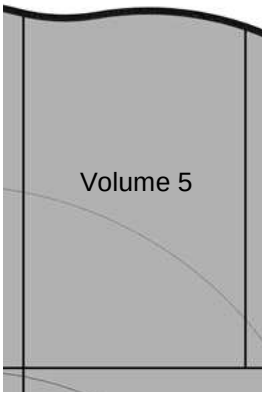
_____

Volume 4 = _____ m^3

Question 11

Calculate Volume 5. First mark on the diagram to the right each of the relevant dimensions from pages 5 and 6.

Volume 5 very closely approximates a rectangle. This will simplify your calculation. Use the same method that you have followed above.

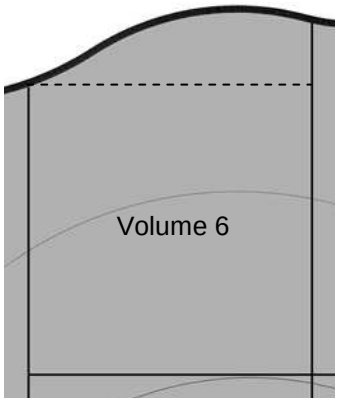


Volume 5 = _____ m^3

Question 12

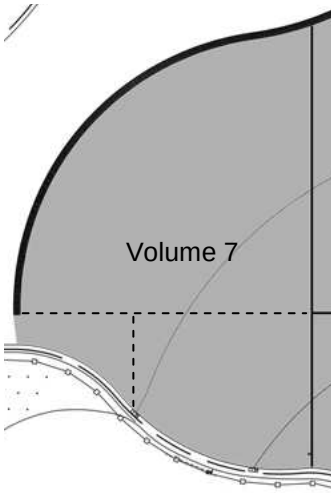
Calculate Volume 6. First mark on the diagram to the right each of the relevant dimensions from pages 5 and 6.

Think of Volume 6 as a rectangle with a section of a circle added, as marked in the diagram. Devise your own method to estimate the area of this addition, then follow the same method that you have used above to calculate the volume.



Volume 6 = _____ m^3

Question 13



Calculate Volume 7. First mark on the diagram to the right each of the relevant dimensions from pages 5 and 6.

Volume 7 can be divided into three sections. The top one can be thought of as a quarter of a cone, the volume of which can be found in the same manner as Volume 4. The other two volumes can be found in a similar manner to Volumes 2 and 6.

Volume 7 = _____ m^3

Question 14

Copy each of your calculated volumes from Questions 7 through 13 to the spaces below, and sum them to find the total volume of the Cave of Waves.

Volume 1 = _____ m^3

Volume 2 = _____ m^3

Volume 3 = _____ m^3

Volume 4 = _____ m^3

Volume 5 = _____ m^3

Volume 6 = _____ m^3

Volume 7 = _____ m^3

Total Volume of the Cave of Waves = _____ m^3

Question 15

Comment on your result and suggest ways of improving the method used.